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Multiplying and Dividing Rational Expressions

Unit 11 Lesson 2

Multiplying and Dividing Rational Expressions

Students will be able to:

Simplify and perform multiplication and division of rational expressions.

Key Vocabulary:

- Fraction
- Common Factor
- Reciprocal
- Multiplying
- Dividing

Multiplying and Dividing Rational Expressions

Multiplying and Dividing Rational Expressions

To ***multiply*** and ***divide*** two fractions, make us of the following *theorems*:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad \text{where } b \neq 0, \quad d \neq 0$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \quad \text{where } b \neq 0, \quad d \neq 0$$

Multiplying and Dividing Rational Expressions

Sample Problem 1: Multiply the following rational expressions.

$$1. \frac{8}{y} \times \frac{y^2}{24}$$

$$2. \frac{3x}{x^2 - 4} \times \frac{x + 2}{6}$$

$$3. \frac{3y^3}{2x(a + b)} \times \frac{4x^3(a + b)}{ay^3 - by^3}$$

$$4. \frac{2x - y}{4x^2 - 4y^2} \times \frac{x^2 + 2xy + y^2}{2x^2 + xy - y^2}$$

Multiplying and Dividing Rational Expressions

Sample Problem 1: Multiply the following rational expressions.

$$1. \frac{8}{y} \times \frac{y^2}{24}$$

Solution:

$$\frac{8}{y} \times \frac{y^2}{24} = \frac{\cancel{8}}{\cancel{y}} \times \frac{y(\cancel{y})}{3(\cancel{8})}$$

$$= \frac{y}{3}$$

$$2. \frac{3x}{x^2 - 4} \times \frac{x + 2}{6}$$

Solution:

$$\frac{3x}{x^2 - 4} \times \frac{x + 2}{6}$$

$$= \frac{\cancel{3}x}{(\cancel{x+2})(x-2)} \times \frac{\cancel{x+2}}{\cancel{3}(2)}$$

$$= \frac{x}{2(x-2)}$$

Multiplying and Dividing Rational Expressions

Sample Problem 1: Multiply the following rational expressions.

$$3. \frac{3y^3}{2x(a+b)} \times \frac{4x^3(a+b)}{ay^3 - by^3}$$

Solution:

$$\frac{3y^3}{2x(a+b)} \times \frac{4x^3(a+b)}{ay^3 - by^3} = \frac{\cancel{3y^3}}{\cancel{2x}(a+b)} \times \frac{\cancel{4x}(x^2)(\cancel{a+b})}{y^3(a-b)} = \frac{6x^2}{a-b}$$

Multiplying and Dividing Rational Expressions

Sample Problem 1: Multiply the following rational expressions.

$$4. \frac{2x - y}{4x^2 - 4y^2} \times \frac{x^2 + 2xy + y^2}{2x^2 + xy - y^2}$$

Solution:

$$\frac{2x - y}{4x^2 - 4y^2} \times \frac{x^2 + 2xy + y^2}{2x^2 + xy - y^2} = \frac{\cancel{2x - y}}{4(\cancel{x + y})(x - y)} \times \frac{(\cancel{x + y})(\cancel{x + y})}{(\cancel{2x - y})(\cancel{x + y})}$$

$$= \frac{1}{4(x - y)}$$

Multiplying and Dividing Rational Expressions

Sample Problem 2: Divide the following rational expressions.

$$5. \frac{7y}{6x} \div \frac{21y}{36x}$$

$$6. \frac{x}{x+5} \div \frac{x}{x+7}$$

$$7. \frac{10ab}{x-3y} \div \frac{-5a^3}{7x-21y}$$

$$8. \frac{x^2-9}{x^2-25} \div \frac{x-3}{x+5}$$

Multiplying and Dividing Rational Expressions

Sample Problem 2: Divide the following rational expressions.

$$5. \frac{7y}{6x} \div \frac{21y}{36x}$$

Solution:

$$\frac{7y}{6x} \div \frac{21y}{36x} = \frac{7y}{6x} \times \frac{36x}{21y} = \frac{\cancel{7}y}{6\cancel{x}} \times \frac{6(\cancel{6}x)}{3(\cancel{7}y)} = \frac{6}{3} = 2$$

Multiplying and Dividing Rational Expressions

Sample Problem 2: Divide the following rational expressions.

$$6. \frac{x}{x+5} \div \frac{x}{x+7}$$

Solution:

$$\frac{x}{x+5} \div \frac{x}{x+7} = \frac{\cancel{x}}{x+5} \times \frac{x+7}{\cancel{x}} = \frac{x+7}{x+5}$$

Multiplying and Dividing Rational Expressions

Sample Problem 2: Divide the following rational expressions.

$$7. \frac{10ab}{x-3y} \div \frac{-5a^3}{7x-21y}$$

Solution:

$$\frac{10ab}{x-3y} \div \frac{-5a^3}{7x-21y} = \frac{10ab}{x-3y} \times \frac{7x-21y}{x-3y}$$

$$= \frac{\cancel{5a}(2b)}{\cancel{x-3y}} \times \frac{7(\cancel{x-3y})}{\cancel{5a}(-a^2)} = -\frac{14b}{a^2}$$

Multiplying and Dividing Rational Expressions

Sample Problem 2: Divide the following rational expressions.

$$8. \frac{x^2 - 9}{x^2 - 25} \div \frac{x - 3}{x + 5}$$

Solution:

$$\frac{x^2 - 9}{x^2 - 25} \div \frac{x - 3}{x + 5} = \frac{x^2 - 9}{x^2 - 25} \times \frac{x + 5}{x - 3}$$

$$= \frac{(x + 3)\cancel{(x - 3)}}{\cancel{(x + 5)}(x - 5)} \times \frac{\cancel{x + 5}}{\cancel{x - 3}} = \frac{x + 3}{x - 5}$$

Multiplying and Dividing Rational Expressions

Sample Problem 3: Perform the indicated operation to the following rational expressions.

$$9. \frac{12x}{x+y} \div \left(\frac{5x-5}{x^2-y^2} \times \frac{3xy}{xy-y} \right)$$

$$10. \frac{x^2-25}{x^2-x-12} \div \frac{x^2-x-20}{3x-3} \times \frac{x^2-16}{x^2+4x-5}$$

Multiplying and Dividing Rational Expressions

Sample Problem 3: Perform the indicated operation to the following rational expressions.

$$9. \frac{12x}{x+y} \div \left(\frac{5x-5}{x^2-y^2} \times \frac{3xy}{xy-y} \right)$$

Solution:

$$\begin{aligned} \frac{12x}{x+y} \div \left(\frac{5x-5}{x^2-y^2} \times \frac{3xy}{xy-y} \right) &= \frac{12x}{x+y} \div \left(\frac{5(\cancel{x-1})}{x^2-y^2} \times \frac{3xy}{y(\cancel{x-1})} \right) \\ &= \frac{12x}{x+y} \div \frac{15x}{x^2-y^2} = \frac{12x}{x+y} \times \frac{x^2-y^2}{15x} = \frac{4(\cancel{3x})}{\cancel{x+y}} \times \frac{(\cancel{x+y})(x-y)}{5(\cancel{3x})} = \frac{4(x-y)}{5} \end{aligned}$$

Multiplying and Dividing Rational Expressions

Sample Problem 3: Perform the indicated operation to the following rational expressions.

$$10. \frac{x^2 - 25}{x^2 - x - 12} \div \frac{x^2 - x - 20}{3x - 3} \times \frac{x^2 - 16}{x^2 + 4x - 5}$$

Solution:

$$\begin{aligned} \frac{x^2 - 25}{x^2 - x - 12} \div \frac{x^2 - x - 20}{3x - 3} \times \frac{x^2 - 16}{x^2 + 4x - 5} &= \frac{x^2 - 25}{x^2 - x - 12} \times \frac{3x - 3}{x^2 - x - 20} \times \frac{x^2 - 16}{x^2 + 4x - 5} \\ &= \frac{\cancel{(x+5)}\cancel{(x-5)}}{\cancel{(x-4)}(x+3)} \times \frac{3\cancel{(x-1)}}{\cancel{(x-5)}\cancel{(x+4)}} \times \frac{\cancel{(x+4)}\cancel{(x-4)}}{\cancel{(x+5)}\cancel{(x-1)}} = \frac{3}{x+3} \end{aligned}$$